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THE DIFFRACTIVE MULTIFOCAL FOCUSING EFFECT AND THE PHASE OF THE OPTICAL FIELD

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Abstract

When a monochromatic plane wave of intensity I_0 is normally incident upon a circular aperture, the intensity at on-axis observation points after the aperture oscillates between the values $4I_0$ and zero as the distance from the aperture is increased. This has been referred to as the diffractive multifocal focusing of radiation (DMFR) effect. The DMFR effect can dramatically increase the on-axis intensity of the diffracted wave, as compared to the incident wave, in two- or multi-circular aperture systems. In this paper the phase of the diffracted field is investigated for both one- and two-aperture systems. For the one aperture system, it is shown that in the neighborhood of a focal point (where the Fresnel number is odd) the phase of the wave approaching the focal point is that of a converging wave, the phase in the focal plane is planar, and the phase of the wave exiting the focal point is that of a diverging wave. It is shown that as the observation point on-axis passes through a singular point (where the intensity is zero and the phase is undefined), the nature of the wave in the neighborhood of the axis changes from that of a diverging wave to that of a converging wave, i.e., the wave refocuses. Similar behavior is observed for the two-aperture system, and the behavior of this system depends on the ratio of the radii of the two apertures.

1 Introduction

It is well known that when a monochromatic plane wave of intensity I_0 is normally incident upon a circular aperture, the intensity at on-axis observation points after the aperture oscillates between the values $4I_0$ and zero as the distance from the aperture is increased. The reason for this is that the various Fresnel zones in the aperture contribute either constructively or destructively to the amplitude of the field at the observation point in question, causing the amplitude to oscillate between zero and twice the incident field value. For an incident wavelength λ, aperture radius a, and aperture-plane to observation-plane distance z, the number of zones that contribute is given by the Fresnel number, $N = a^2/\lambda z$. The maxima and minima occur, respectively, at observation points where the Fresnel number is an odd or even integer.

What is generally not appreciated is the fact that in the region near the axis, as z is increased, the light is repeatedly focusing, then defocusing, and then refocusing, over and over again, due to diffraction. These focal points occur at positions where the Fresnel number is an odd integer. This was pointed out in a series of papers by Lit and co-workers [1-3] and most recently by Letfullin and George, [4] who referred to this phenomenon as the diffractive multifocal focusing of radiation (DMFR) effect.

Letfullin and George used the DMFR effect to propose a system of two circular apertures that would increase the on-axis intensity of an incident monochromatic plane wave dramatically. In their system the second aperture was located where the Fresnel number of the first aperture was unity. They analyzed this system theoretically, and showed that the on-axis intensity after the second aperture oscillates between maximum values of the order of ten times that of the incident wave and minimum values that were small, but nonzero. These predictions were verified experimentally [5],[6] and extended theoretically to incident fields with a Gaussian amplitude distribution [7].

In this paper we substantiate the focusing, defocusing, and refocusing interpretation mentioned above by investigating the phase of the diffracted field created when a monochromatic plane wave is normally incident upon a circular aperture, and we use these results to explain the behavior of the bicomponent system of Ref. 4. In particular, it is shown that in the neighborhood of a focal point (where the Fresnel number is odd) the phase of the wave approaching the focal point is that of a converging wave, the phase in the focal plane is planar, and the phase of the wave exiting the focal point is that of a diverging wave. It is also shown that the wave becomes more and more divergent as the distance from the focal point is increased, until a position is reached where the Fresnel number is even. At such a point the intensity of the wave is zero, and the phase of the wave is undefined, i.e., singular. It is shown that as the observation point on-axis moves away from the aperture and passes through a singular point, the nature of the wave in the neighborhood of the axis changes from that of a diverging wave to that of a converging wave, i.e., the wave refocuses.

In Sec. 2 we will investigate the intensity and phase of the field diffracted by a single circular aperture. In Sec. 3 we use the results of Sec. 2 to investigate the intensity and the phase of the field after the second aperture in a system of two circular apertures. We use scalar wave theory throughout. Unlike in Ref. 4, where the monochromatic wave equation (i.e., the Helmholtz equation) was integrated numerically, we use the Fresnel approximation and the paraxial approximation in our calculations.

2 Intensity and Phase of the Field Diffracted by a Circular Aperture

A Basic Equations

Consider a monochromatic plane wave of amplitude U_0 and frequency ω, propagating in the positive z direction, and normally incident upon an opaque screen in the plane $z=0$, containing an aperture of radius a. The aperture is centered about the origin. Let $P'=(x',y',0)$ be a point inside the aperture, and let $P=(x,y,z)$ be a point in an observation plane z = constant > 0 (see Fig. 1). In cylindrical polar coordinates we have $P'=(\rho',\theta',0)$ and $P=(\rho,\theta,z)$. We assume a time dependence of $\exp(-i\omega t)$ for the incident field. The complex amplitude, $U^{(i)}(\rho,\theta,z)$, of the incident field is given by

$$U^{(i)}(\rho,\theta,z)=U_0 e^{ikz}, \tag{2.1}$$

where U_0 is a positive constant and $k=\omega/c$ is the wave number of the light.

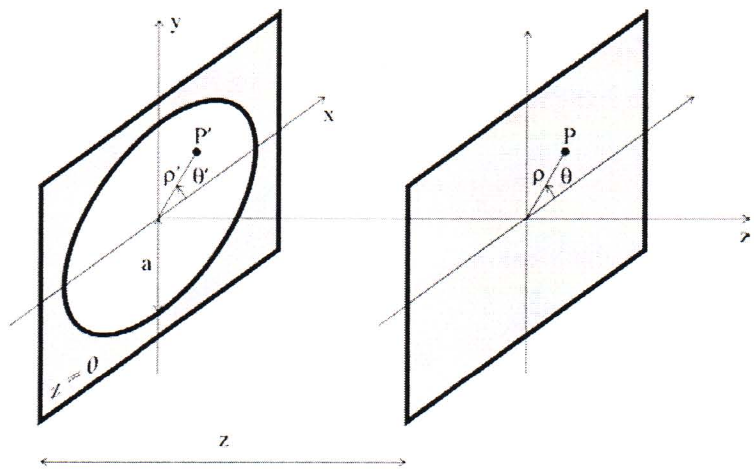

Fig. 1. Geometry for the diffraction of a plane wave from a circular aperture with radius a. Point $P'=(\rho',\theta',0)$ is a point inside the aperture and $P=(\rho,\theta,z)$ is a point in the observation plane z = constant.

We make the following assumptions. First, that the wavelength λ is much smaller than the distance z from the aperture plane to the observation plane. Second, that the Fresnel number of the observation plane is small and that the transverse distance ρ is less than the aperture radius a and much less than the distance z. In this case the paraxial form of the Fresnel approximation to the Rayleigh-Sommerfeld diffraction formula [8] is appropriate for describing the field, and the complex amplitude, $U(\rho,\theta,z)$, of the diffracted field can be written as

$$U(\rho,\theta,z)=\frac{k}{2\pi iz}U_0 e^{ikz}e^{ik\rho^2/2z}\int_0^a\int_0^{2\pi}U^{(i)}(\rho',\theta',0)\,e^{ik\rho'^2/2z}e^{-ik\rho\rho'\cos(\theta-\theta')/z}\rho' d\rho' d\theta'. \quad (2.2)$$

Equation (2.2) is equivalent to the formula used by Lommel9 for the case in which the incident field is a diverging spherical wave.

Let us now simplify this equation. Upon substituting the value of $U(\rho',\theta',0)$ from Eq. (2.1) into this equation and performing the angular integration, we find that the complex amplitude of the field is independent of the angle θ and is given by the expression

$$U(\rho,z)=\frac{k}{iz}U_0 e^{ikz}e^{ik\rho^2/2z}\int_0^a e^{ik\rho'^2/2z}J_0(k\rho\rho'/z)\rho' d\rho', \quad (2.3)$$

where $J_0(x)$ is the zero order Bessel function of the first kind. Let us now make the change of variables $\rho'=\xi a$. Upon making this change, Eq. (2.3) can be rewritten in terms of two dimensionless variables u and v as

$$U(\rho,z)=-iuU_0 e^{ikz}e^{iv^2/2u}\int_0^1 e^{iu\xi^2/2}J_0(v\xi)\xi d\xi, \quad (2.4)$$

where

$$u=2\pi N, \qquad v=2\pi N\rho/a, \quad (2.5)$$

and N is the Fresnel number of the aperture at the on-axis observation point,

$$N=a^2/\lambda z. \quad (2.6)$$

Let us now put Eq. (2.4) into a form more suitable for calculations. It is shown in the Appendix that in the lit region (where $\rho<a$ and hence $v<u$) the integral on the right-hand side of Eq. (2.4) can be expressed in as

$$\int_0^1 e^{iu\xi^2/2}J_0(v\xi)\xi d\xi=\frac{i}{u}\left\{e^{-iv^2/2u}-e^{iu/2}\left[V_0(u,v)-iV_1(u,v)\right]\right\}, \quad (2.7)$$

where $V_0(u,v)$ and $V_1(u,v)$ are Lommel functions of two variables:

$$V_n(u,v) = \sum_{s=0}^{\infty}(-1)^s\left(\frac{v}{u}\right)^{2s+n} J_{2s+n}(v). \tag{2.8}$$

Upon substituting the right-hand side of Eq. (2.7) into Eq.(2.4), the field in the lit region attains the form

$$U(\rho,z) = U_0\, e^{ikz}\, F(\rho,z), \tag{2.9}$$

where

$$F(\rho,z) = 1 - e^{iu/2} e^{iv^2/2u}\left[V_0(u,v) - iV_1(u,v)\right]. \tag{2.10}$$

The intensity, $I(\rho,z)$, of the field in the lit region is given by

$$I(\rho,z) = U(\rho,z)^* U(\rho,z) = I_0\left|F(\rho,z)\right|^2, \tag{2.11}$$

where * denotes the complex conjugate and $I_0 = |U_0|^2$ is the intensity of the incident field. It follows from Eq. (2.9) that the phase $\phi(\rho,z)$ of the field is given by

$$\phi(\rho,z) = kz + \psi(\rho,z), \tag{2.12}$$

where

$$\psi(\rho,z) = \arg F(\rho,z), \tag{2.13}$$

and arg denotes the argument of the complex-valued function $F(\rho,z)$. We shall refer to $\psi(\rho,z)$ as the reduced phase.

B Discussion of the Results

In this section the intensity and reduced phase in observation planes at a variety of distances from the aperture will be investigated. For the sake of comparison, let us first recall the paraxial form for a diverging spherical wave. A spherical wave emanating from the origin and arriving at the position P in Fig. 1 is described by the wave function $\exp(ikr)/r$ where $r = \sqrt{\rho^2 + z^2}$. The paraxial approximation to this function is

$\exp(ikr)/r \approx \exp\left[ik\left(z+\rho^2/2z\right)\right]/z$. Upon comparing this equation to Eq. (2.9), we see that the paraxial approximation to the reduced phase of this wave is

$$\psi(\rho,z) \approx k\rho^2/2z. \tag{2.14}$$

The intensity and reduced phase of the diffracted field along the x axis in several observation planes at different distances from the aperture plane are plotted in Figs. 2 and 3. The behavior of the field as we travel outward from the aperture plane and pass through a focal point is depicted in Figs. 2(a) through 2(c). In Fig. 2(a) the Fresnel number is 3.5, and in this plane we see that the reduced phase near the axis has a curvature with the opposite sign of that of the phase in Eq. (2.14). Hence the wave in this plane corresponds to a converging wave. The on-axis intensity value is approximately 2.0. In Fig. 2(b) the Fresnel number is 3. This is a focal plane, and we see that the reduced phase is constant near the axis, i.e., the wave is behaving like a plane wave in this region. The on-axis intensity value in this case is 4.0. In Fig. 2(c) the Fresnel number is 2.5, and we see that in this plane the reduced phase near the axis has a curvature with the same sign as the phase in Eq. (2.14). Hence the wave corresponds to a diverging wave. The on-axis intensity value is approximately 1.2.

As we move further away from the aperture plane, the wave near the axis diverges more strongly, until we reach the on-axis point where the Fresnel number is 2.0. At this point the intensity of the field is zero, and its phase is undefined. Such a point is referred to as a singular point of the field. [10],[11] Figure 2(d) shows the intensity and phase of the wave in the plane where the Fresnel number is 2.01, i.e., just before we reach the singular point. The phase near the axis has a steep upward curvature, corresponding to a strongly diverging wave. The value of the phase on-axis is approximately $-\pi/2$. Figure 2(e) shows the intensity and phase of the wave in the plane where the Fresnel number is 1.99, i.e., just after we have passed through the singular point. The phase near the axis has a steep downward curvature, corresponding to a strongly converging wave. The on-axis value of the phase here is approximately $\pi/2$, so as we pass through the singular point, the phase jumps by π. Figure 2(f) shows that as we continue to move further away from the aperture plane, the field near the axis starts to converge less strongly. Finally, at the next focal point, thc plane where $N = 1.0$, we see that the phase near the axis is again constant; and hence the wave is again behaving like a plane wave. This is shown in Fig. 3. In addition, by comparing Fig. 3 to Fig. 2(b), we see that for focal points further from the aperture plane, the wavefront is planar over a larger area in the observation plane.

Figure 4 shows the lines of constant phase, ϕ, in the xz plane near the $N = 2$ singular point, for the case $a/\lambda = 50$. It is evident from the figure that as the wave approaches the singular point, it is a diverging wave whose radius of curvature becomes smaller and smaller. It also follows from the figure that after the wave has passed through the singular point, it is now a *converging* wave whose radius of curvature is increasing.

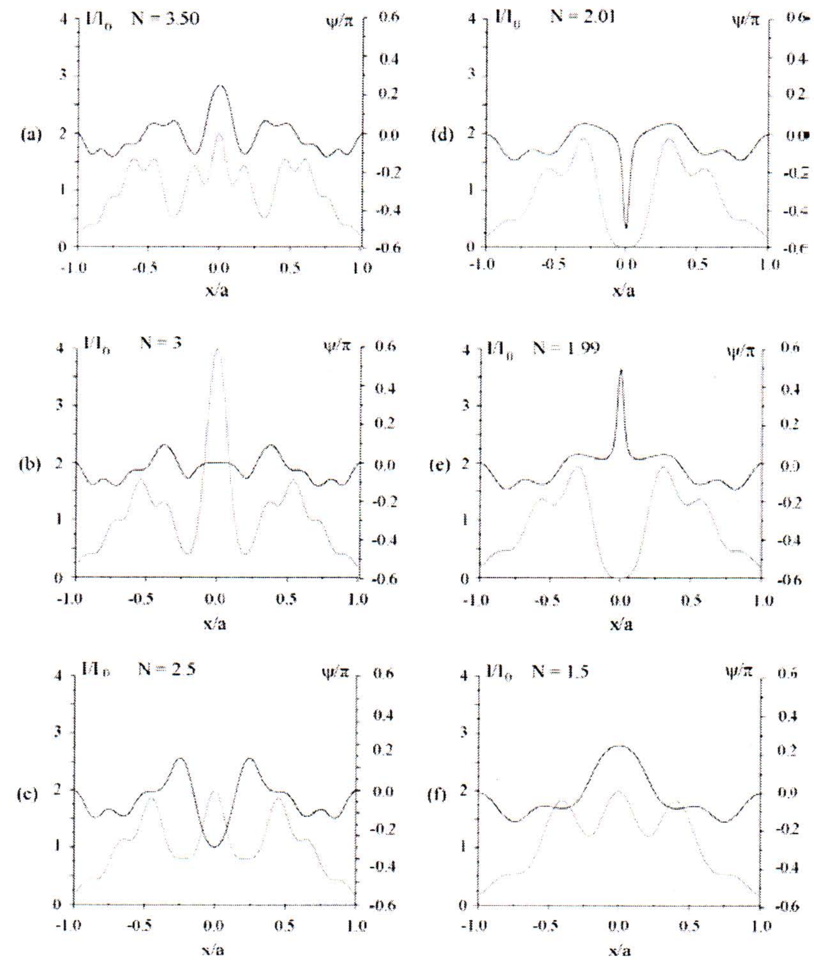

Fig. 2. Plots of I/I_0 (thin line) and ψ/π (thick line) as functions of position along the x axis for observation planes with Fresnel numbers: (a) $N = 3.5$, (b) $N = 3$, (c) $N = 2.5$, (d) $N = 2.01$, (e) $N = 1.99$, and (f) $N = 1.5$.

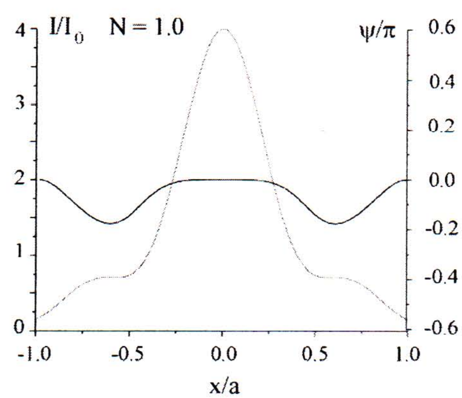

Fig. 3. Plots of I/I_0 (thin line) and ψ/π (thick line) as functions of position along the x axis for the observation plane with $N = 1$.

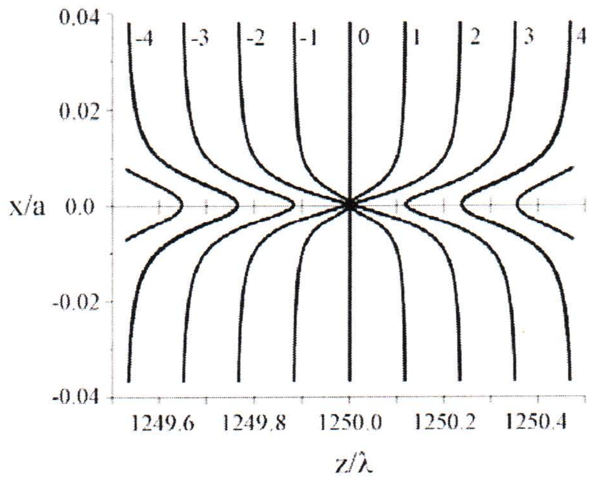

Fig. 4. Lines of constant phase, ϕ, in the xz plane near the $N = 2$ singular point when $a/\lambda = 50$. The transverse coordinate x is in units of a, and the z coordinate is in units of the wavelength λ. The value of the phase for each line is labeled in units of $\pi/4$.

3 Intensity and Phase of the Field Diffracted by a Bicomponent System of Apertures

A Basic Equations

Let us now consider the two-aperture system depicted in Fig. 5. The radius of the first aperture is a_1, the radius of the second aperture is a_2, and the distance between the planes containing the apertures is L. Let $P'' = (x'', y'', L)$ be a point inside the second aperture, and $P = (x, y, z)$ be an observation point in the plane z = constant > L. In cylindrical polar coordinates we then write $P'' = (\rho'', \theta'', L)$ and $P = (\rho, \theta, z)$. It follows from Eqs. (2.9) and (2.10) that the complex amplitude of the field incident upon the second aperture is given by

$$U(\rho'', \theta'', L) = U_0 e^{ikL} \left\{1 - e^{iu_1/2} e^{iv_1^2/2u_1} \left[V_0(u_1, v_1) - iV_1(u_1, v_1)\right]\right\}, \tag{3.1}$$

where $u_1 = 2\pi N_1$, $v_1 = 2\pi N_1 \rho''/a_1$, and $N_1 = a_1^2/\lambda L$.

In order to compare our results to those of Ref. 4, we now assume that the distance L is such that N_1, the Fresnel number of the first aperture at the center of the second aperture, is equal to unity. In this case $u_1 = 2\pi$, $v_1 = 2\pi\rho''/a_1$, and Eq. (3.1) can be written as

$$U(\rho'', \theta'', L) = U_0 e^{ikL} \left[1 + D(\rho''/a_1)\right], \tag{3.2}$$

where

$$D(w)=e^{i\pi w^2}\left[V_0(2\pi,2\pi w)-iV_1(2\pi,2\pi w)\right]. \tag{3.3}$$

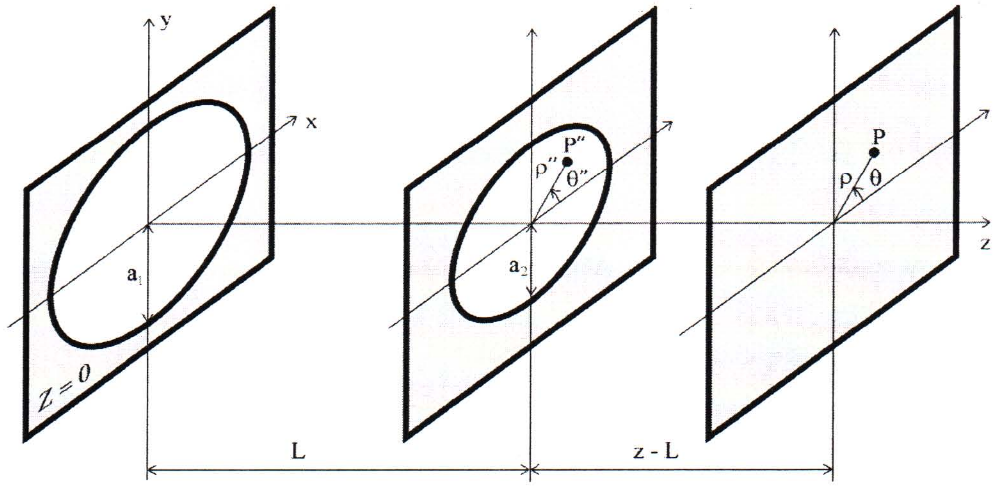

Fig. 5. Geometry for the diffraction of a plane wave by a system of two circular apertures with radii a_1 and a_2, respectively. The distance between the two aperture planes is L. Point $P''=(\rho'',\theta'',L)$ is a point inside the second aperture, and $P=(\rho,\theta,z)$ is a point in the observation plane z = constant.

Let us now investigate the complex amplitude, $U(\rho,\theta,z)$, of the diffracted field in the region $z > L$. The paraxial form of the Fresnel approximation to the Rayleigh-Sommerfeld diffraction formula tells us that this field is given by the expression

$$U(\rho,\theta,z)=\frac{k}{2\pi i(z-L)}e^{ik(z-L)}e^{\frac{ik\rho^2}{2(z-L)}}\int_0^{a_2}\int_0^{2\pi}U(\rho'',\theta'',L)e^{\frac{ik\rho''^2}{2(z-L)}}e^{-ik\rho\rho''\cos(\theta-\theta'')/(z-L)}\rho''d\rho''d\theta''. \tag{3.4}$$

Upon substituting the right-hand side of Eq. (3.2) into Eq. (3.4) and performing the angular integration, we find that the field is independent of angle θ and is given by

$$U(\rho,z)=\frac{k}{i(z-L)}U_0e^{ikz}e^{\frac{ik\rho^2}{2(z-L)}}\int_0^{a_2}\left[1+D(\rho''/a_1)\right]e^{\frac{ik\rho''^2}{2(z-L)}}J_0\left[\frac{k\rho\rho''}{z-L}\right]\rho''d\rho''. \tag{3.5}$$

Let us now make the change of variable $\rho''=\xi a_2$. After using this relation in the right-hand side of Eq.(3.5), we find that the field can be described in terms of the dimensionless variables u_2 and v_2 as

$$U(\rho,z) = -iu_2 U_0 e^{ikz} e^{iv_2^2/2u_2} \int_0^1 \left[1 + D(\alpha\xi)\right] e^{iu_2\xi^2/2} J_0(v_2\xi)\xi d\xi, \tag{3.6}$$

where

$$u_2 = 2\pi N_2, \qquad v_2 = 2\pi N_2 \rho''/a_2, \tag{3.7}$$

with N_2 the Fresnel number of the second aperture at the on-axis point in the observation plane,

$$N_2 = a_2^2/\lambda(z-L), \tag{3.8}$$

and α the ratio of the radii of the two apertures,

$$\alpha = a_2/a_1. \tag{3.9}$$

By analogy with the results of Sec. 2, let us write Eq. (3.6) as

$$U(\rho,z) = U_0 e^{ikz} F(\rho,z), \tag{3.10}$$

where

$$F(\rho,z) = -iu_2 e^{iv_2^2/2u_2} \int_0^1 \left[1 + D(\alpha\xi)\right] e^{iu_2\xi^2/2} J_0(v_2\xi)\xi d\xi. \tag{3.11}$$

It follows from Eq. (3.10) that the intensity and phase of the field in the lit region are given, respectively, by Eqs. (2.11) and (2.12) with $F(\rho,z)$ given by Eq. (3.11) and the reduced phase defined as in Eq. (2.13). The function $F(\rho,z)$ can be evaluated by numerical integration.

B Discussion of the Results

The on-axis intensity and reduced phase of the field after the second aperture were calculated by the method described above, and are plotted as a function of the Fresnel number N_2 for two different values of α in Figs. 6 and 7. One general comment is in order before discussing the results. There are no true singular points after the second aperture, because even at points where the intensity is minimum, its value is not exactly zero.

Figure 6 shows the results for $\alpha = 0.1$. The value of the reduced phase oscillates and undergoes a phase change of approximately 0.8π at each point where the Fresnel number is even. The value of the intensity oscillates and is approximately zero at each point where the

Fresnel number is even and $15I_0$ at each focal point (where the Fresnel number is odd). Figure 7 shows the results for α = 0.5. The value of the reduced phase oscillates with smaller amplitude than in the previous case, and the value of the intensity oscillates between the values I_0 and $8I_0$. This figure shows, especially at the higher values of N_2, that the maxima and minima no longer occur exactly at integer values of the Fresnel number.

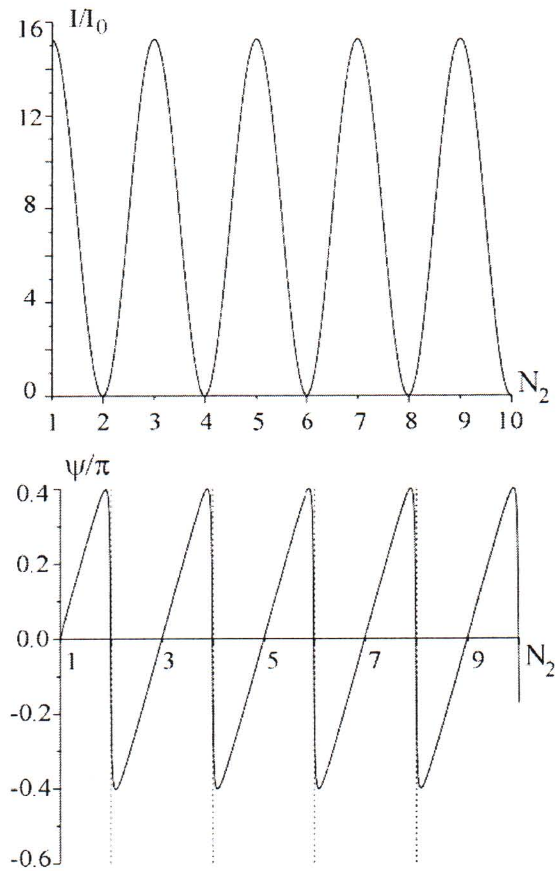

Fig. 6. Plots of I/I_0 and ψ/π on-axis as functions of the Fresnel number N_2 for α = 0.1.

The explanation for this behavior is as follows. When α = 0.1, the radius of the second aperture is ten times smaller than that of the first. In this case the results of Sec. 2 show that the phase of the field incident upon the second aperture is constant across it (see Fig. 3), and that the value of the intensity incident upon it varies by only 10% across it. Therefore the field incident upon the second aperture is very similar to the field incident upon the first aperture (a constant amplitude, normally incident plane wave), and this explains why the second aperture increases the on-axis intensity at the focal points by a factor of close to four, why the intensity at singular points is approximately zero, and why the phase jump at a singular point is close to π. As α increases from the value 0.1, the phase of the field incident upon the second aperture remains approximately constant across it, but the intensity begins to vary considerably. As a result, the effect of the second aperture becomes less ideal. When α = 0.5, as in Fig. 7, both the phase and the intensity of the field incident upon the second aperture vary significantly across it (see Fig. 3), and the effect of the second aperture is correspondingly less ideal.

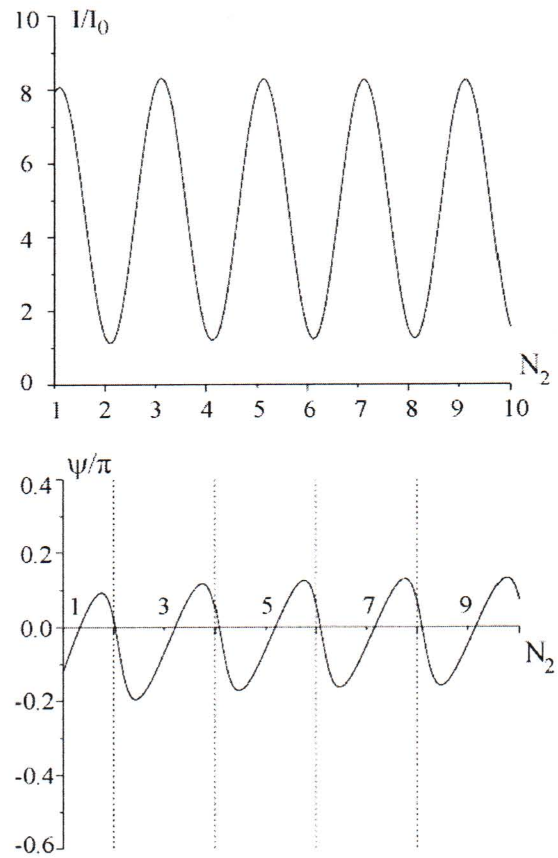

Fig. 7. Plots of I/I_0 and ψ/π for on-axis observation positions as functions of the Fresnel number N_2 for $\alpha = 0.5$.

The intensity and reduced phase of the diffracted field as a function of the scaled transverse coordinate x/a_2 in six different planes after the second aperture are shown in Fig. 8 for the case $\alpha = 0.4$. In Fig. 8(a) the Fresnel number is 3, and the plane is a focal plane. The on-axis intensity is maximum, and the phase near the axis is approximately constant. In Figs. 8(b) and 8(c) the Fresnel numbers are 2.5 and 2.2, and the wave is diverging in each case. In Fig. 8(d) the Fresnel number is 1.85, and the wave has changed from a diverging wave to a converging wave. In Fig. 8(d) the Fresnel number is 1.5, and the wave continues to converge. In Fig. 8(a) the Fresnel number is 1, and the plane is a focal plane. The on-axis intensity is maximum, and the phase near the axis is approximately constant. In Ref. 4 the value of α was 0.4, as it is in Fig. 8. Our results for the intensities agree well with those of Ref. 4.

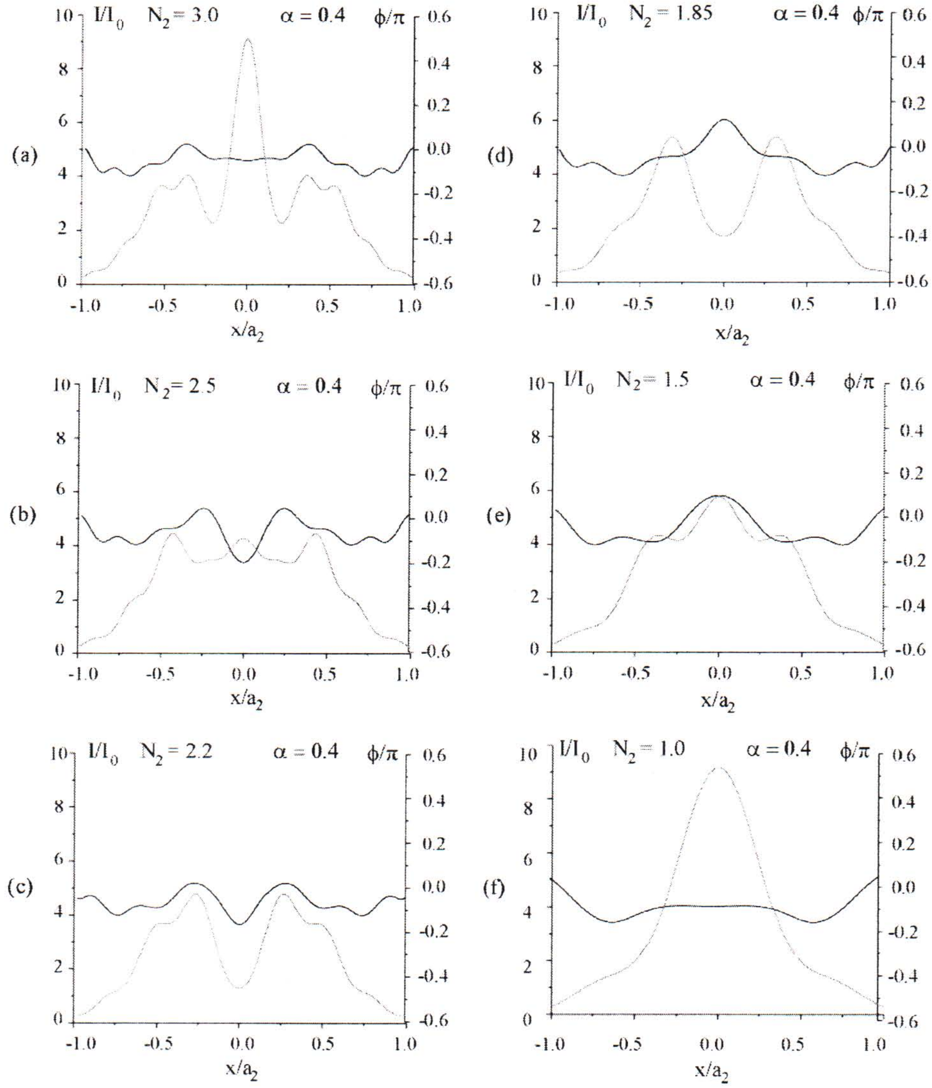

Fig. 8. Plots of I/I_0 (thin line) and ψ/π (thick line) as functions of position along the x axis for observation planes after the second aperture with Fresnel numbers: (a) N_2 = 3.0, (b) N_2 = 2.5, (c) N_2 = 2.2, (d) N_2 = 1.85, (e) N_2 = 1.5, and (f) N_2 = 1.0.

4 Conclusions

We have investigated for the first time the phase of the diffracted wave for both one- and two-aperture systems. The paraxial and Fresnel approximations to the Rayleigh-Sommerfeld diffraction theory have been used to investigate the phase and amplitude of the diffracted field for a monochromatic plane wave normally incident upon a circular aperture. For the one aperture system it was shown that in the neighborhood of a focal point (where the Fresnel number is odd) the phase of the wave approaching the focal point is that of a converging wave, the phase in the focal plane is planar, and the phase of the wave exiting the focal point is that of a diverging wave. It was also shown that the wave becomes more and more divergent as the distance from the focal point is increased, until a position at which the Fresnel number is even is reached. At such a point the intensity of the wave is zero, and the phase of the wave is undefined, i.e., singular. It was shown that as the observation point on-axis moves away from the aperture and passes through a singular point, the nature of the wave in the neighborhood of the axis changes from that of a diverging wave to that of a converging wave, i.e., the wave refocuses. It was also shown that similar behavior is observed for the two-aperture system, and that the behavior of this system depends on the ratio of the radii of the two apertures.

Appendix

It is convenient to consider the real and imaginary parts of the integral on the left-hand side of Eq. (2.7) separately. We set

$$\int_0^1 e^{iu\xi^2/2} J_0(v\xi)\xi d\xi = \tfrac{1}{2}\left[C(u,v) + iS(u,v)\right], \tag{A1}$$

where

$$C(u,v) = 2\int_0^1 \cos(u\xi^2/2) J_0(v\xi)\xi d\xi \tag{A2}$$

$$S(u,v) = 2\int_0^1 \sin(u\xi^2/2) J_0(v\xi)\xi d\xi \tag{A3}$$

These two integrals can be expressed in terms of the Lommel functions of two variables $V_0(u,v)$ and $V_1(u,v)$, [12]

$$C(u,v) = \frac{2}{u}\left[\sin\left(v^2/2u\right) + \sin(u/2)V_0(u,v) - \cos(u/2)V_1(u,v)\right], \tag{A4}$$

$$S(u,v) = \frac{2}{u}\left[\cos\left(v^2/2u\right) - \cos(u/2)V_0(u,v) - \sin(u/2)V_1(u,v)\right]. \tag{A5}$$

Upon substituting the right-hand sides of Eqs. (A4) and (A5) into Eq. (A1), we find

$$\int_0^1 e^{iu\xi^2/2} J_0(v\xi)\xi d\xi = \frac{i}{u}\left\{e^{-iv^2/2u} - e^{iu/2}\left[V_0(u,v) - iV_1(u,v)\right]\right\}. \tag{A6}$$

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